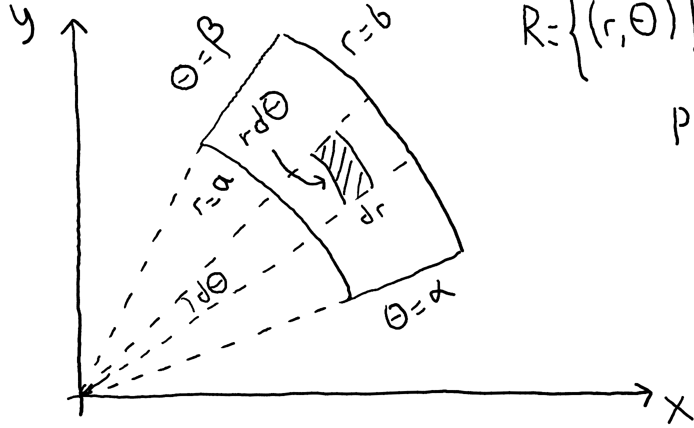
Lecture 15

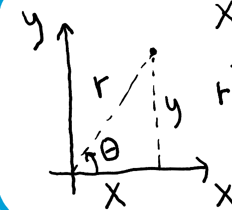
Friday, 10 April 2020 00:47

Double Integrals in Polar Coordinates



$R = \{(r,\theta) \mid a \leq r \leq b,\ \alpha \leq \theta \leq \beta\}$

polar rectangle

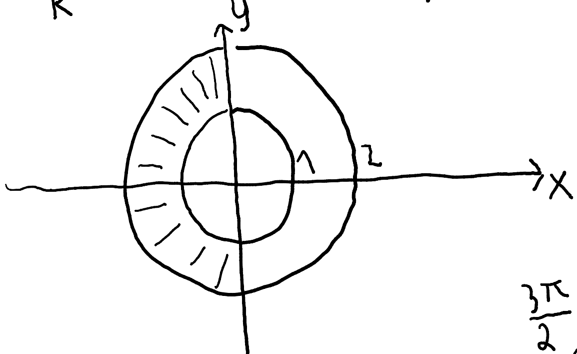


$x = r\cos\theta \quad y = r\sin\theta$

$r^2 = x^2 + y^2 \quad \tan\theta = \frac{y}{x}$

$$\iint_R f(x,y)\,dA = \int_\alpha^\beta \int_a^b f(r\cos\theta, r\sin\theta)\, r\,dr\,d\theta$$

Ex: $\iint_R (x+y)\,dA$ where R is the region that lies to the left of the y-axis between the circles $x^2+y^2=1$ and $x^2+y^2=4$.



$$\int_{\pi/2}^{3\pi/2} \int_1^2 (r\cos\theta + r\sin\theta)\, r\,dr\,d\theta = \int_{\pi/2}^{3\pi/2} (\cos\theta + \sin\theta)\, \frac{1}{3} r^3 \Big|_1^2 \,d\theta$$

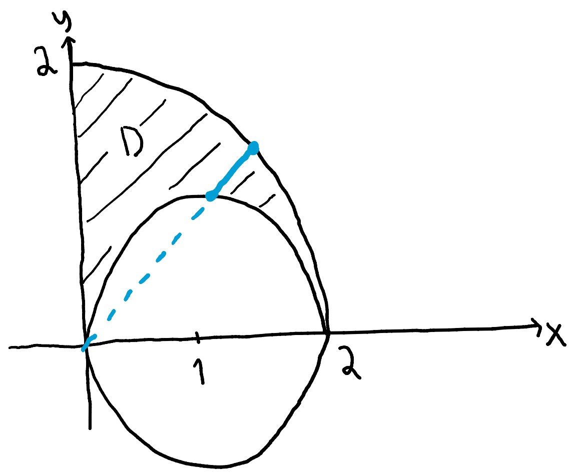
$$\ldots (\sin\theta - \cos\theta) \Big|^{3\pi/2} = \frac{7}{3}(-2).$$

$$= \frac{1}{3} 7 \left(\sin\theta - \cos\theta\right)\Big|_{\pi/2}^{-\pi/2} = \frac{7}{3}(-2).$$

Ex: Find the volume of the solid under the paraboloid $z = x^2 + y^2$ and above the disk $x^2 + y^2 \leq 9$.

$$\iint_{x^2+y^2\leq 9} (x^2+y^2)\, dA = \int_0^{2\pi}\int_0^3 r^2 r \, dr \, d\theta = \theta\Big|_0^{2\pi} \frac{1}{4} r^4 \Big|_0^3 = \frac{81}{2}\pi$$

Ex: $\iint_D x \, dA$ D is the region in the first quadrant that lies between the circles $x^2+y^2=4$ and $x^2+y^2=2x$.



$\underbrace{x^2 + y^2 = 2x}$ $\Longleftrightarrow (x-1)^2 + y^2 = 1$

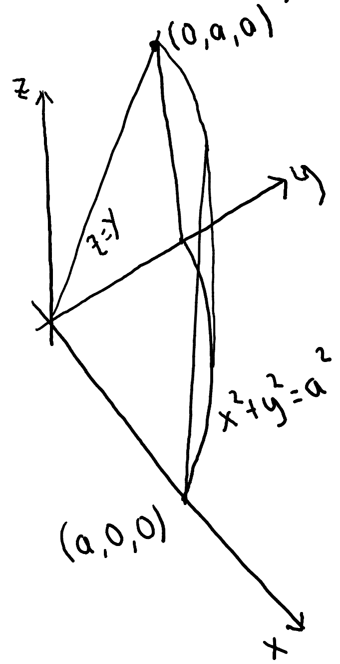
polar equation: $r^2 = 2r\cos\theta$

$r = 2\cos\theta$

$$\int_0^{\frac{\pi}{2}} \int_{2\cos\theta}^{2} r\cos\theta \, r \, dr \, d\theta$$

Ex: Find the volume of the solid lying in the first octant, inside the cylinder $x^2+y^2=a^2$, and under the plane $z=y$.

Ex: Find the volume of the solid lying in the [illegible] inside the cylinder $x^2+y^2=a^2$, and under the plane $z=y$.



$$\int_0^{\frac{\pi}{2}} \int_0^{a} (r\sin\theta)\, r\, dr\, d\theta = \frac{1}{3}a^3$$