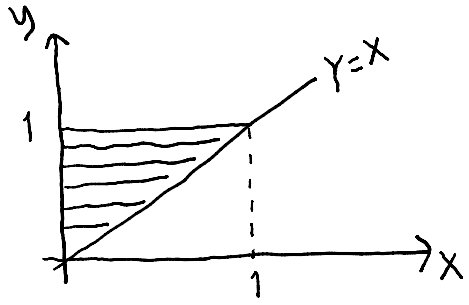
Lecture 13
Monday, 6 April 2020 22:59

Properties of Double Integrals

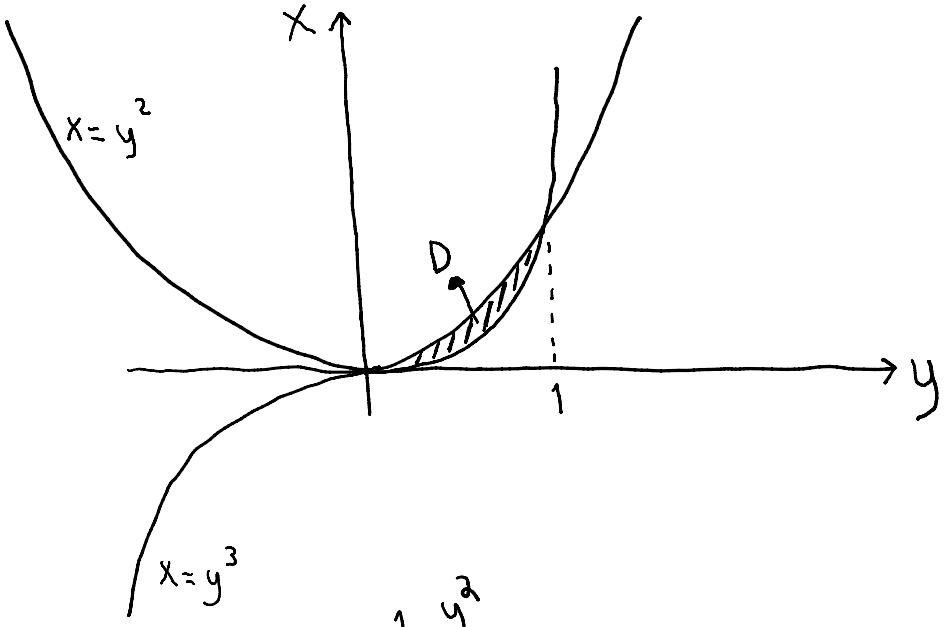
- $\iint_D [cf(x,y) + g(x,y)]\,dA = c\iint_D f(x,y)\,dA + \iint_D g(x,y)\,dA$
- $f(x,y) \geq g(x,y)$ for all (x,y) in $D \Rightarrow \iint_D f(x,y)\,dA \geq \iint_D g(x,y)\,dA$
- $\iint_D 1\,dA = A(D)$: area of D
- $m \leq f(x,y) \leq M$ for all (x,y) in $D \Rightarrow mA(D) \leq \iint_D f(x,y)\,dA \leq MA(D)$
- $\left| \iint_D f(x,y)\,dA \right| \leq \iint_D |f(x,y)|\,dA$

Ex: $\int_0^1 \int_x^1 \sin(y^2)\,dy\,dx$



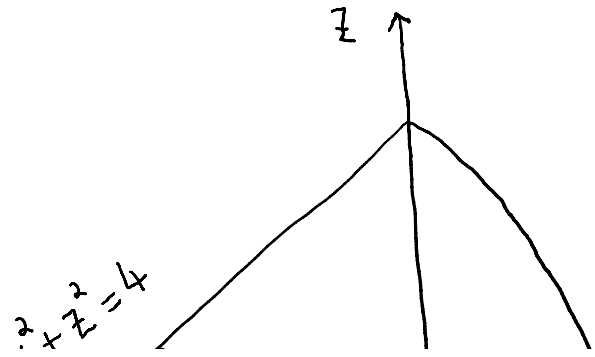
$$= \int_0^1 \int_0^y \sin(y^2)\,dx\,dy = \int_0^1 y\sin(y^2)\,dy = -\frac{1}{2}\cos(y^2)\Big|_0^1$$

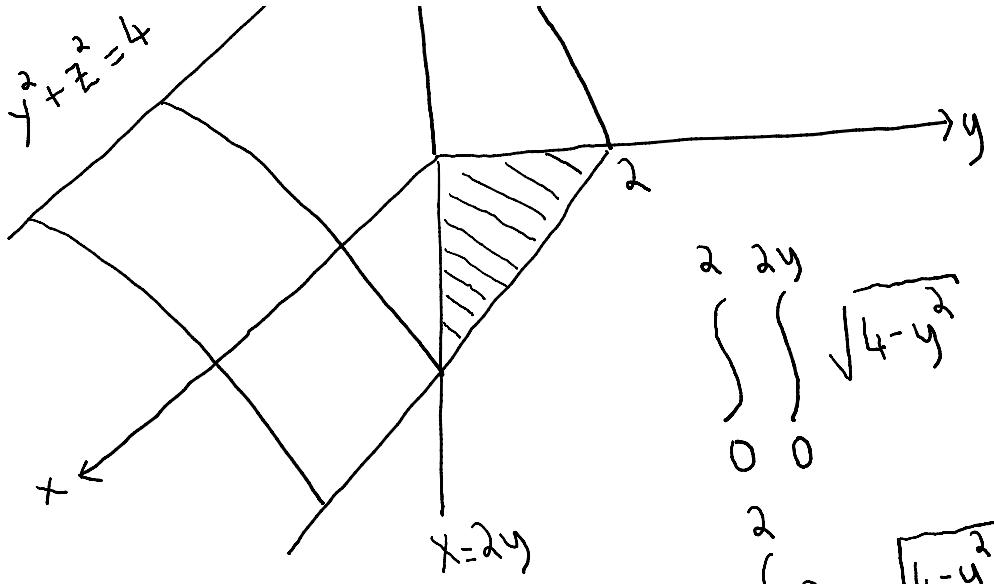
Ex: Find the volume under the surface $z = 2x + y^2$ and above the region bounded by $x = y^2$ and $x = y^3$.



$$\iint_D (2x + y^2)\,dA = \int_0^1 \int_{y^3}^{y^2} (2x + y^2)\,dx\,dy$$

Ex: Find the volume of the solid that is bounded by the cylinder $y^2 + z^2 = 4$ and the planes $x = 2y$, $x = 0$, $z = 0$ in the first octant.

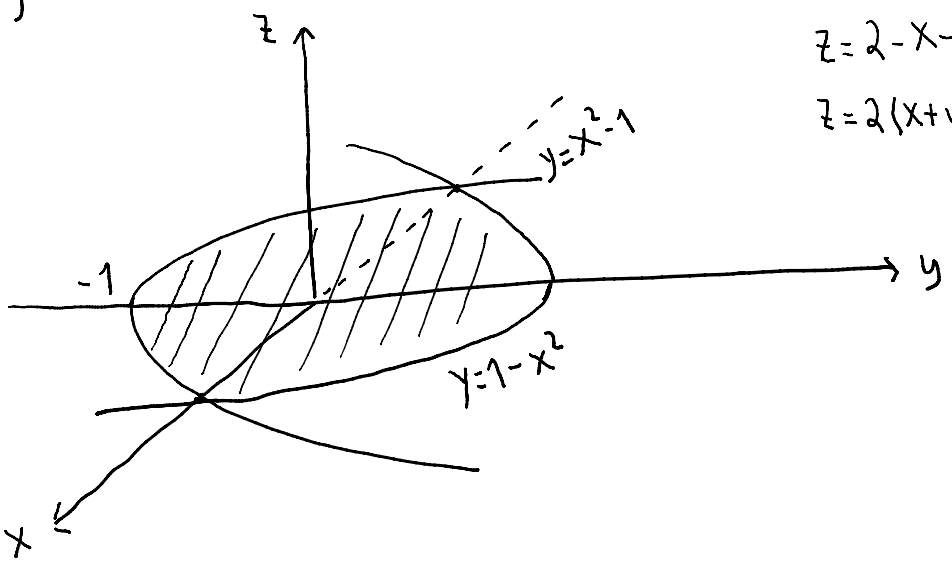




$$\int_0^2 \int_0^{2y} \sqrt{4-y^2}\, dx\, dy$$

$$= \int_0^2 2y\sqrt{4-y^2}\, dy$$

Ex: Find the volume of the solid that is enclosed by the parabolic cylinders $y=1-x^2$, $y=x^2-1$ and the planes $x+y+z=2$, $2x+2y-z+10=0$.



$$V = \int_{-1}^{1} \int_{x^2-1}^{1-x^2} \left[(2x+2y+10)-(2-x-y)\right] dy\, dx$$