Math 118, Recitation for Chapter 10.4

Q1) Find the equation of the line...

a) That passes through $(1,-1,10)$ and paralel through the line..

$$L_1 : x+2 = y/2 = z-3$$

Recall $\dfrac{x-a}{v_1} = \dfrac{y-b}{v_2} = \dfrac{z-c}{v_3}$ is an equation of the line that passes through (a,b,c) and that has the direction vector $v := (v_1, v_2, v_3)$

$\Longrightarrow$ So direction vector of L_1 is $\underline{(1, 2, 1)}$

Our line is paralel to $L_1 \Rightarrow$ Then they have the "<u>same direction vector</u>".

$\Rightarrow$ So, we know the <u>point</u> and <u>direction vector</u> of the line.

By the information on "recall" we can write it's equation ..

$$L : \frac{x-1}{1} = \frac{y+1}{2} = \frac{x-10}{1}$$ $\Leftarrow$ This is the desired equation!

b) That passes through $(-1, 0, 1)$ and perpendicular to the plane:

$P_1 : 2x - y + 7z = 12$

"Recall" $ax + by + cz = d$ gives a plane and it's normal vector is given by $(a, b, c) = \vec{v}$

$\Rightarrow$ If our line is perpendicular to the given plane then normal vector of the plane should be a multiple of the direction vector of the line.

$\Rightarrow n_{P_1} = (2, -1, 7) \to$ normal vector of P_1

$\Rightarrow v_L = (2, -1, 7) \to$ direction vector of the line L.

Then as in part a) the line is given by

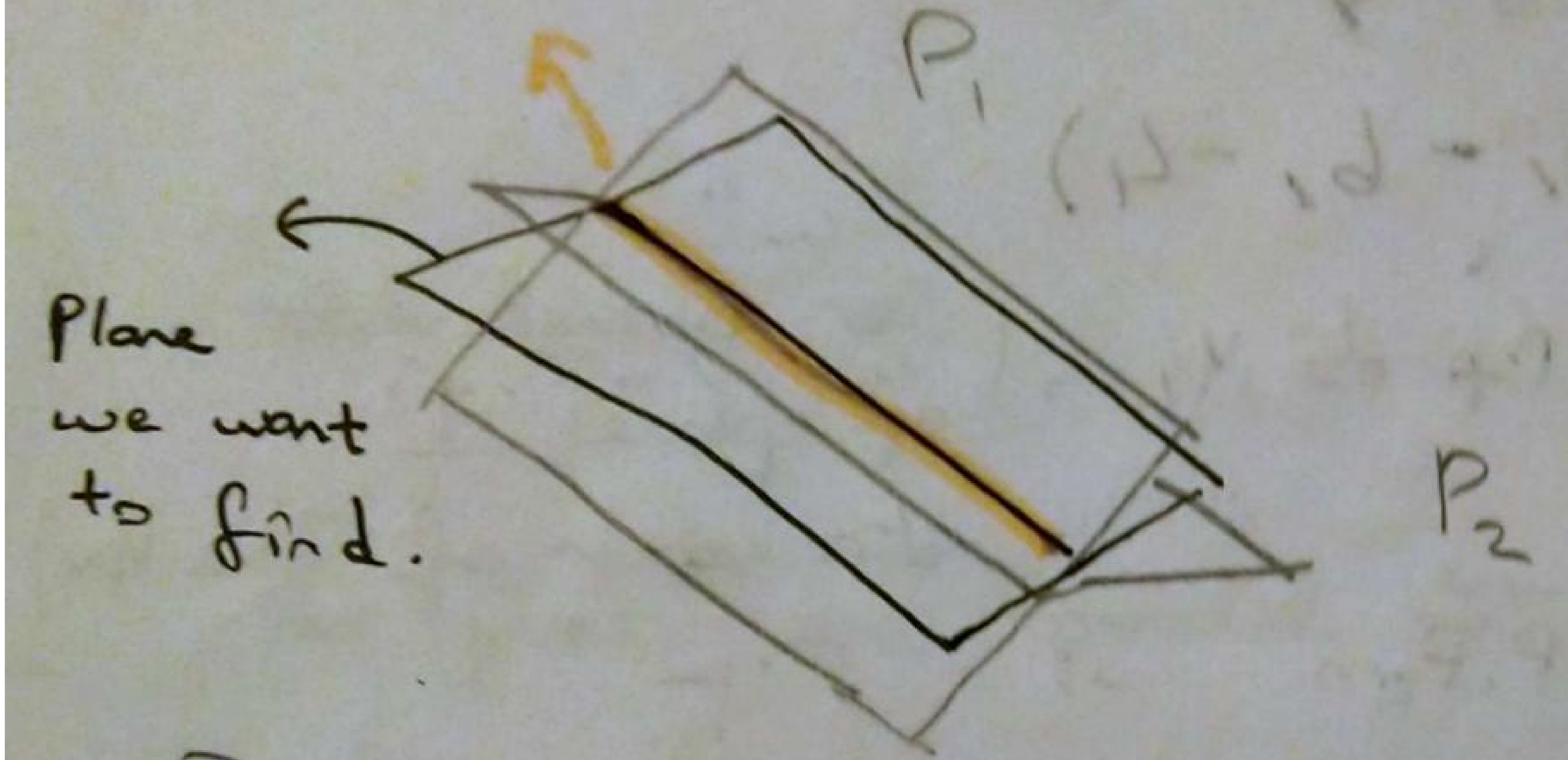
$$L : \frac{x+1}{2} = \frac{y}{-1} = \frac{z-1}{7}$$

Q2) Find the equation of the plane..

a) that passes through $(-1,2,1)$ and contains the line of intersection of planes P_1, P_2 where

$P_1: x+y-z=2$ and $P_2: 2x-y+3z=1$

line of intersection "L"



$P_1 + P_2 \rightarrow 3x+2z=3 \Longrightarrow x = \dfrac{3-2z}{3}$

$3P_1 + P_2 \longrightarrow 5x+2y=7 \quad x = \dfrac{7-2y}{5}$

So the eqn. of line of intersection is given by...

$$L: x = \frac{7-2y}{5} = \frac{3-2z}{3}$$

$\Longrightarrow$ So it's normal is

$\left(1, \frac{-5}{2}, \frac{-3}{2}\right)$ or $(2,-5,-3)$

Q3) Show that the lines L_1, L_2 are skew and find the distance between them - where

$L_1: x = 1 + 2t_1,\ y = 2 - t_1,\ z = 4 + 3t_1$

$L_2: x = 3 + 3t_2,\ y = 6 - 4t_2,\ z = 1 + 5t_2$

$v_{L_1} = (2, -1, 3) \quad v_{L_2} = (3, -4, 5)$

Since L_1, L_2 have different direction vectors they can't be paralel!

So they are skew $\Longleftrightarrow L_1 \cap L_2 = \phi$

Suppose $(x, y, z) \in L_1 \cap L_2$.

$\Rightarrow x = 1 + 2t_1,\ x = 3 + 3t_2 \Rightarrow$ (1) $1 + 2t_1 = 3 + 3t_2$

Similarly (2) $2 - t_1 = 6 - 4t_2$, (3) $4 + 3t_1 = 1 + 5t_2$

(1) $\Rightarrow t_1 = 1 + \frac{3}{2}t_2$, Plug t_1 into (2)

$\Rightarrow 2 - t_1 = 6 - 4t_2 \Rightarrow 1 + \frac{3}{2}t_2 = 6 - 4t_2$

$\Rightarrow t_2 = \frac{10}{11} \Rightarrow t_1 = \frac{-8}{22}$.

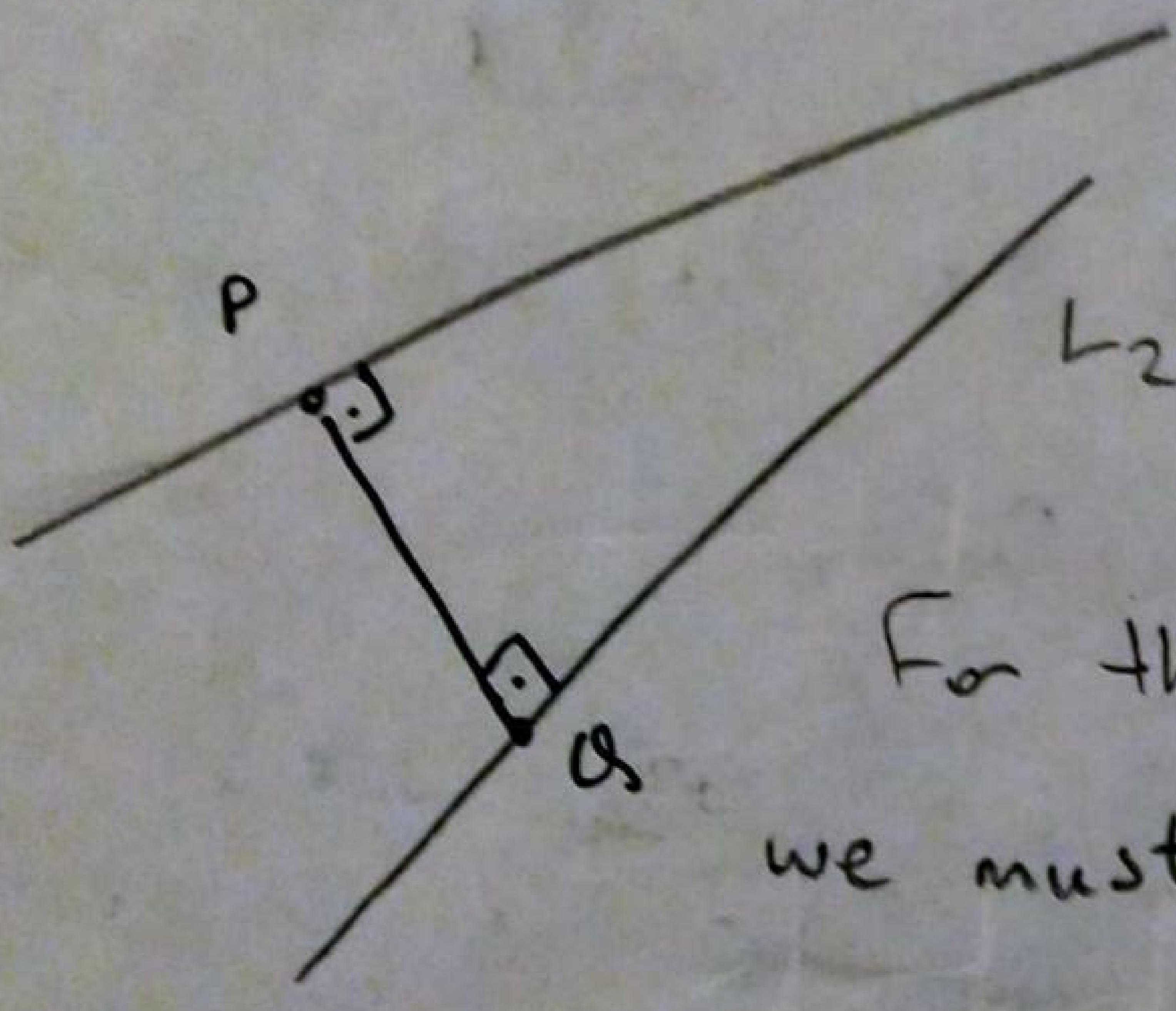
Check that the values

$t_1 = -\frac{8}{22}$, $t_2 = \frac{10}{11}$ does not satisfy equation (3).

$\Rightarrow$ Then we get a contradiction!

$(x,y,z) \notin L_1 \cap L_2 \Rightarrow L_1 \cap L_2 = \phi$ since (x,y,z) was arbitrary.

$\Rightarrow L_1 \cap L_2 = \phi \Longrightarrow$ They are skew!



$L_1 = (1,2,4) + t_1(2,-1,3)$

$L_2 = (3,6,1) + t_2(3,-4,5)$

For the shortest distance we must have $\overrightarrow{PQ} \perp v_{L_1}$

$\overrightarrow{PQ} \perp v_{L_2}$

$$\overrightarrow{PQ} = \begin{pmatrix} 2 - 2t_1 + 3t_2 \\ 4 + t_1 - 4t_2 \\ -3 - 3t_1 + 5t_2 \end{pmatrix} = \lambda \underline{v_{L_1} \times v_{L_2}} = \lambda \begin{pmatrix} 7 \\ -1 \\ -5 \end{pmatrix}$$

Check!

We got three equation

(1) $7\lambda + 2t_1 - 3t_2 = 2$

(2) $-\lambda - t_1 + 4t_2 = 4$

(3) $-5\lambda + 3t_1 - 5t_2 = 3$

(1) $+ 2.$ (2) $\Longrightarrow 5\lambda + 5t_2 = 10$

$\lambda + t_2 = 2$

$\Longrightarrow t_2 = 2 - \lambda$

$5.$ (2) $+ 4.$ (3) $\Rightarrow -25\lambda + 7t_1 = 32$

$\Rightarrow t_1 = \dfrac{32 + 25\lambda}{7}$

Plug t_1, t_2 into eqn (2) $\Longrightarrow$

$-\lambda - \frac{32}{7} - \frac{25}{7}\lambda + 8 - 4\lambda = 4$

$\Rightarrow -7\lambda - 32 - 25\lambda + 56 - 28\lambda = 28$

$\Rightarrow \lambda = -1/15$

distance between L_1, L_2 $= |\overrightarrow{PQ}| = \left| \left(-\frac{1}{15}\right) \begin{pmatrix} 7 \\ -1 \\ -5 \end{pmatrix} \right| = \frac{1}{\sqrt{3}}$