

Spring 2020, Calculus II, Video Lecture Series for online education
due to the COVID-19-Pandemic covering Chapter 10 by Senra Öztürk.

10 Vectors and Coordinate Geometry in 3-Space

10.1 Analytic Geometry in Three Dimensions

Euclidean n -Space
Describing Sets in the Plane, 3-Space, and n -Space

10.2 Vectors

Vectors in 3-Space
~~Hanging Cables and Chains~~
The Dot Product and Projections
Vectors in n -Space

10.3 The Cross Product in 3-Space

Determinants
The Cross Product as a Determinant
Applications of Cross Products

10.4 Planes and Lines

Planes in 3-Space
Lines in 3-Space
Distances

10.5 Quadric Surfaces

Textbook: Calculus
by R. A. Adams & C. Essex

ma118.math.metu.edu.tr
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Why do we need to study $\mathbb{R}^n$ for $n \geq 3$?

$f: D \longrightarrow \mathbb{R}$ where $D \subseteq \mathbb{R}^1$, $f(x) = x^2, \ln(x), \cos(x)$

Next, functions of several variables: (Chap. 12-14)

$f: D \longrightarrow \mathbb{R}$ where $D \subseteq \mathbb{R}^n$, $n \geq 2$

Examples of Functions of two variables

1) $f: \mathbb{R}^2 \longrightarrow \mathbb{R}$ given by $f(x,y) = x^2 + xy + 1$

2) $f: D \longrightarrow \mathbb{R}$ given by $f(x,y) = \sqrt{1-x^2-y^2} + \sin(x) + \cos(xy)$

where $D = \{(x,y) \mid x^2 + y^2 \leq 1\} \subsetneq \mathbb{R}^2$

Examples of functions of three variables

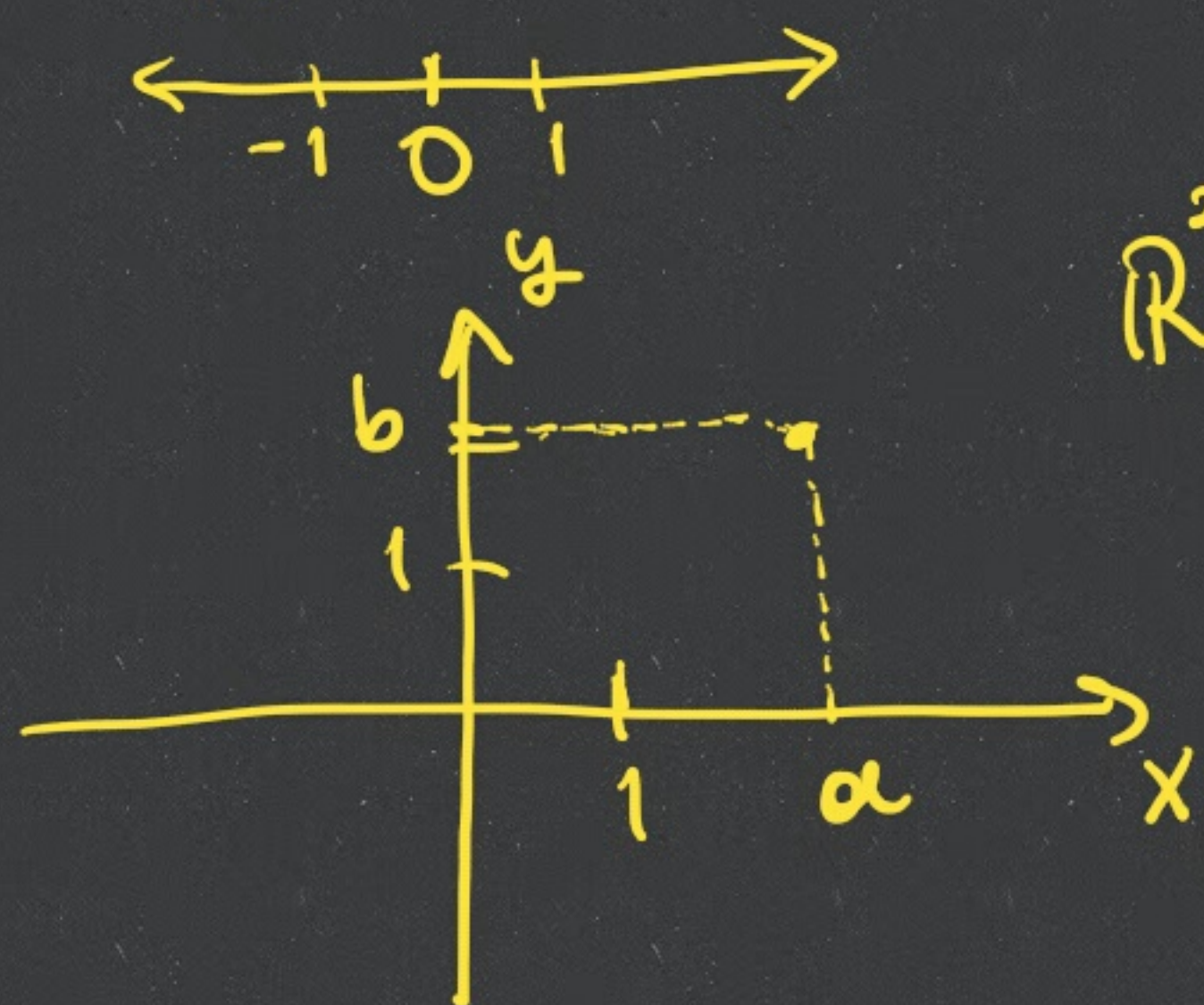
1) $f: \mathbb{R}^3 \longrightarrow \mathbb{R}$ given by $f(x,y,z) = x^2 + y^2 + z^2$

2) $f: D \longrightarrow \mathbb{R}$ given by $f(x,y,z) = x^2 + \sqrt{xy} + z^2$

where $D = \{(x,y,z) \mid xy \geq 0\} \subsetneq \mathbb{R}^3$

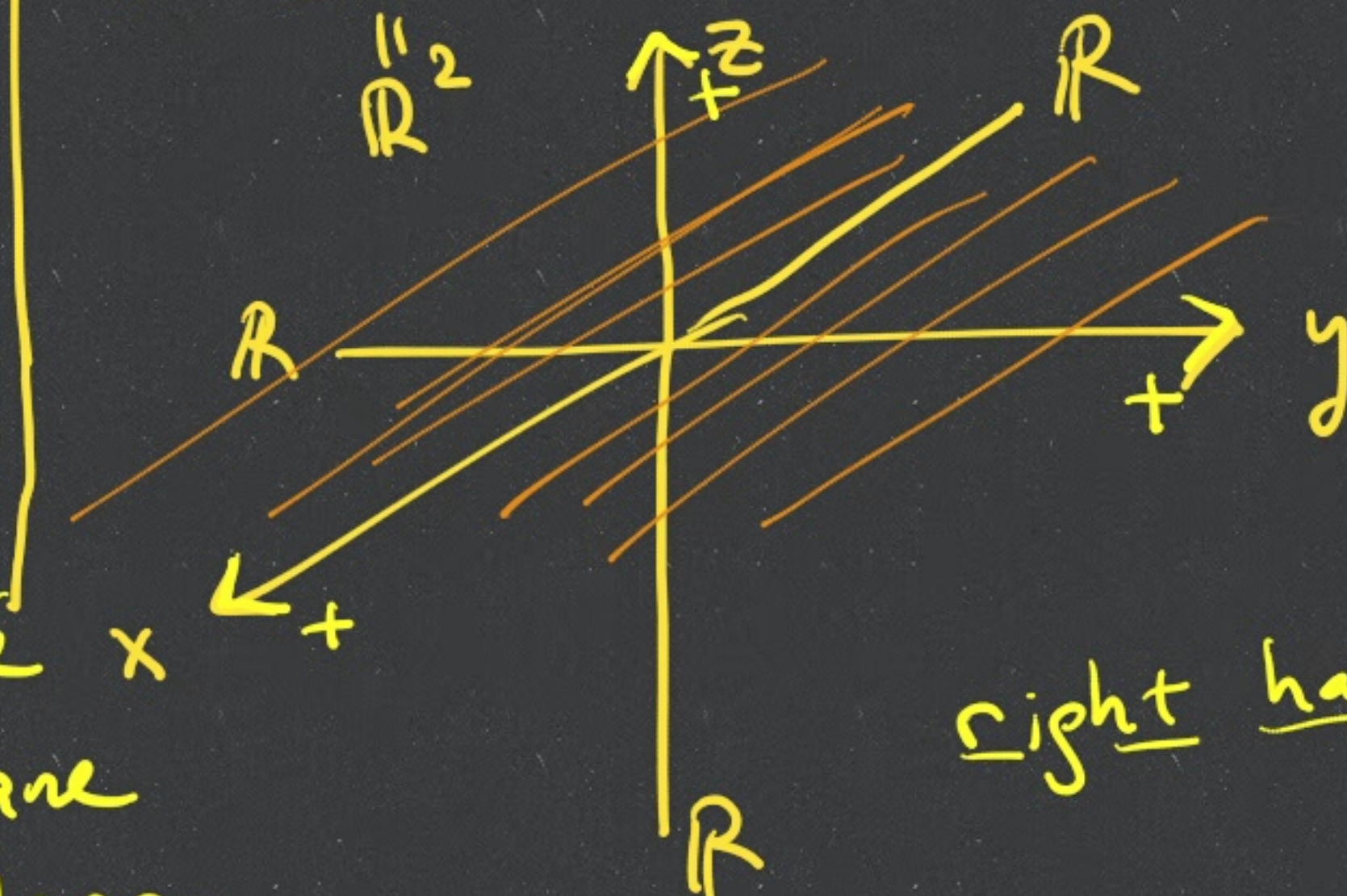
10.1. Analytic geometry in three dimensions.

origin in $\mathbb{R}, \mathbb{R}^2, \mathbb{R}^3$
 $\left\{ \begin{array}{l} xy\text{-plane} \text{ ----} \\ yz\text{-plane} \\ xz\text{-plane} \\ \text{right handed orientation} \end{array} \right.$



$$\mathbb{R}^2 = \mathbb{R} \times \mathbb{R} = \{(a, b) \mid a \in \mathbb{R}, b \in \mathbb{R}\}$$

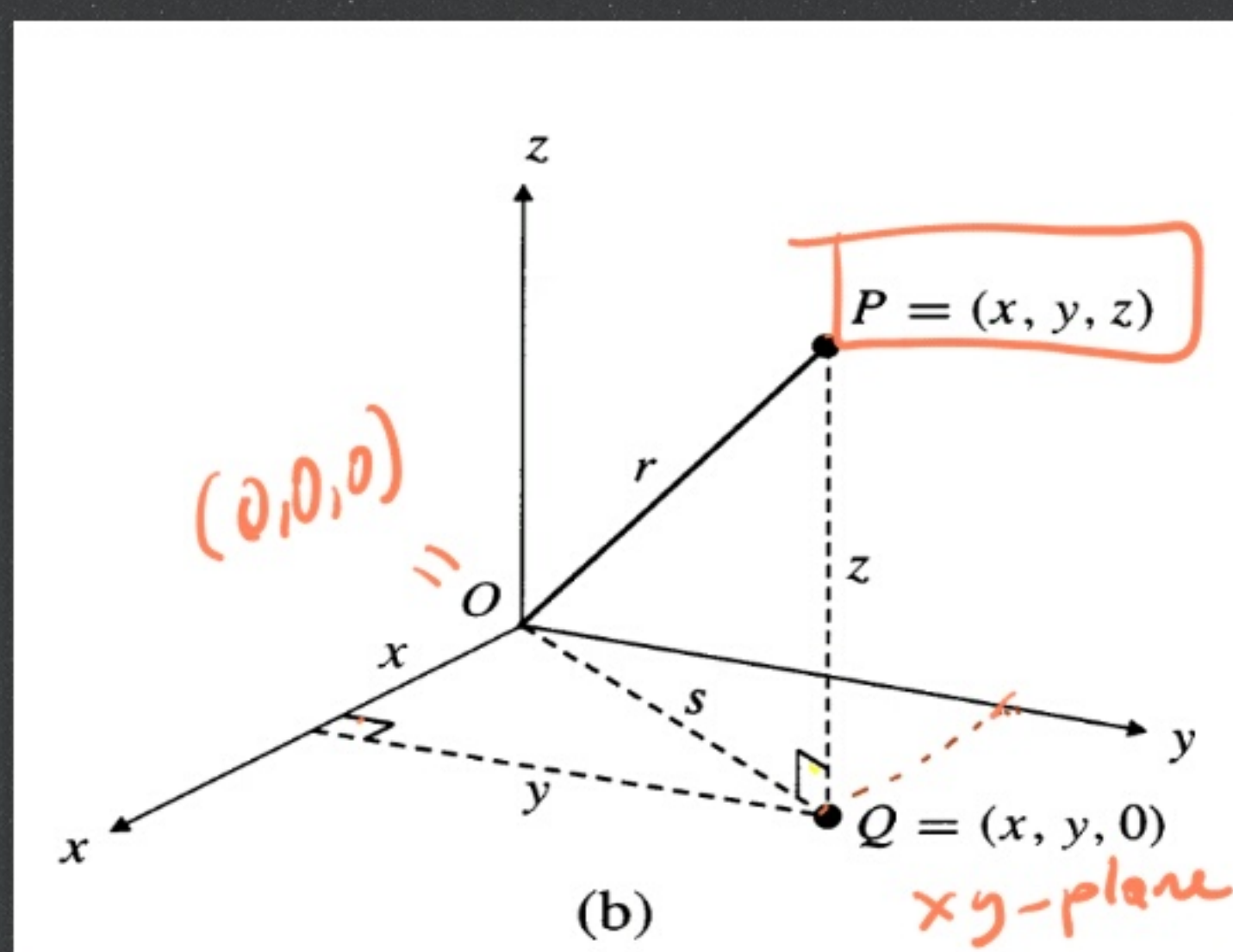
$$\mathbb{R}^3 = [\underbrace{\mathbb{R} \times \mathbb{R}}_{\mathbb{R}^2}] \times \mathbb{R} = \{(a, b, c) \mid a, b, c \in \mathbb{R}\}$$



$z=0$ is the xy -plane
 $y=0$ is the xz -plane
 $x=0$ is the yz -plane

right handed orientation

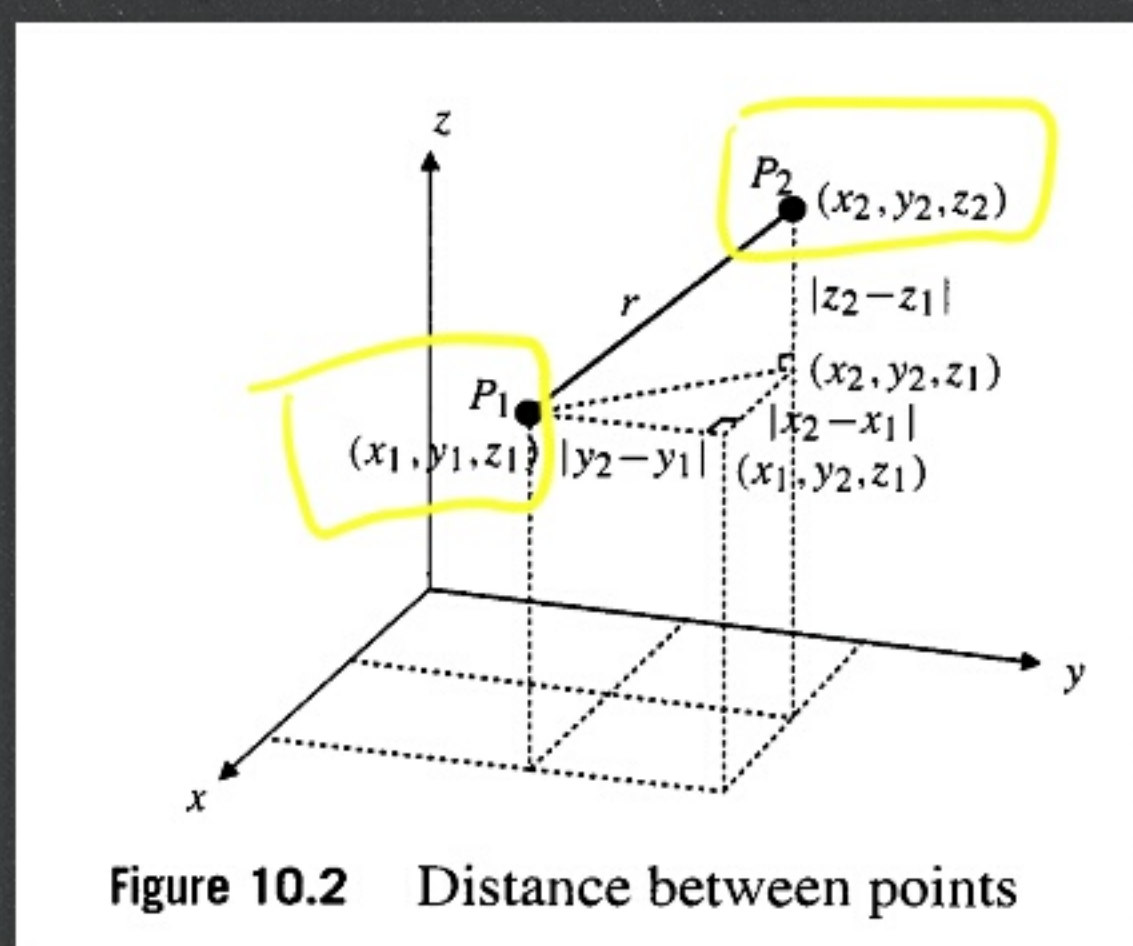
$r =$ Distance of a point $P = (x, y, z)$ to the origin O
 $(0, 0, 0)$



$$s^2 = x^2 + y^2$$

$$r^2 = s^2 + z^2 \Rightarrow r^2 = x^2 + y^2 + z^2$$

$$\Rightarrow r = \sqrt{x^2 + y^2 + z^2}$$



Distance between two points in $\mathbb{R}^3$
 $r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$ is the
 distance between $P_1 = (x_1, y_1, z_1)$, $P_2 = (x_2, y_2, z_2)$.

$$r^2 = (x-a)^2 + (y-b)^2 + (z-c)^2 \quad (*)$$

$(x, y, z) \in \text{Sphere} \Rightarrow (x, y, z)$ satisfies $(*)$

Equation of a sphere of radius (r) centered at (a, b, c) .