Spring 2020, Calculus II, Video Lecture Series for online education due to the CORONA-Pandemic covering Chapter 10 by Semra Öztürk.

10 Vectors and Coordinate Geometry in 3-Space

Textbook: Calculus
by R. A. Adams & C. Essex.

ma118. math. metu. edu.tr
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Why do we need to study $\mathbb{R}^n$ for $n \geq 3$?

$f: D \longrightarrow \mathbb{R}$ where $\boxed{D \subseteq \mathbb{R}^1}$, $f(x) = x^2$, $\ln(x)$, $\cos(x)$

Next, functions of several variables: (Chp. 12-14)

$f: D \longrightarrow \mathbb{R}$ where $D \subseteq \mathbb{R}^n$, $n \geq 2$

Examples of functions of two variables

1) $f: \mathbb{R}^2 \longrightarrow \mathbb{R}$ given by $f(x,y) = x^2 + xy + 1$

2) $f: D \longrightarrow \mathbb{R}$ given by $f(x,y) = \sqrt{1 - x^2 - y^2} + \sin(x) + \cos(xy)$

where $D = \{(x,y) \mid x^2 + y^2 \leq 1\} \subsetneq \mathbb{R}^2$

Examples of functions of three variables

1) $f: \mathbb{R}^3 \longrightarrow \mathbb{R}$ given by $f(x,y,z) = x^2 + y^2 + z^2$

2) $f: D \longrightarrow \mathbb{R}$ given by $f(x,y,z) = x^2 + \sqrt{xy} + z^2$

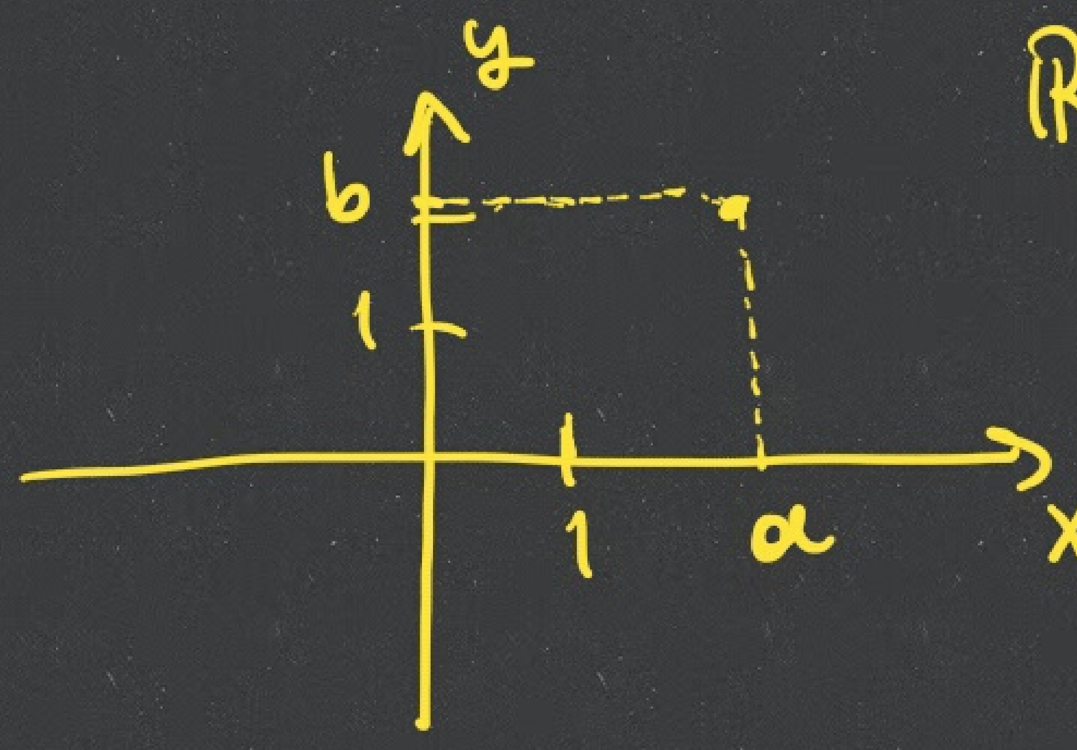
where $D = \{(x,y,z) \mid xy \geq 0\} \subsetneq \mathbb{R}^3$.

10.1. Analytic geometry in three dimensions.

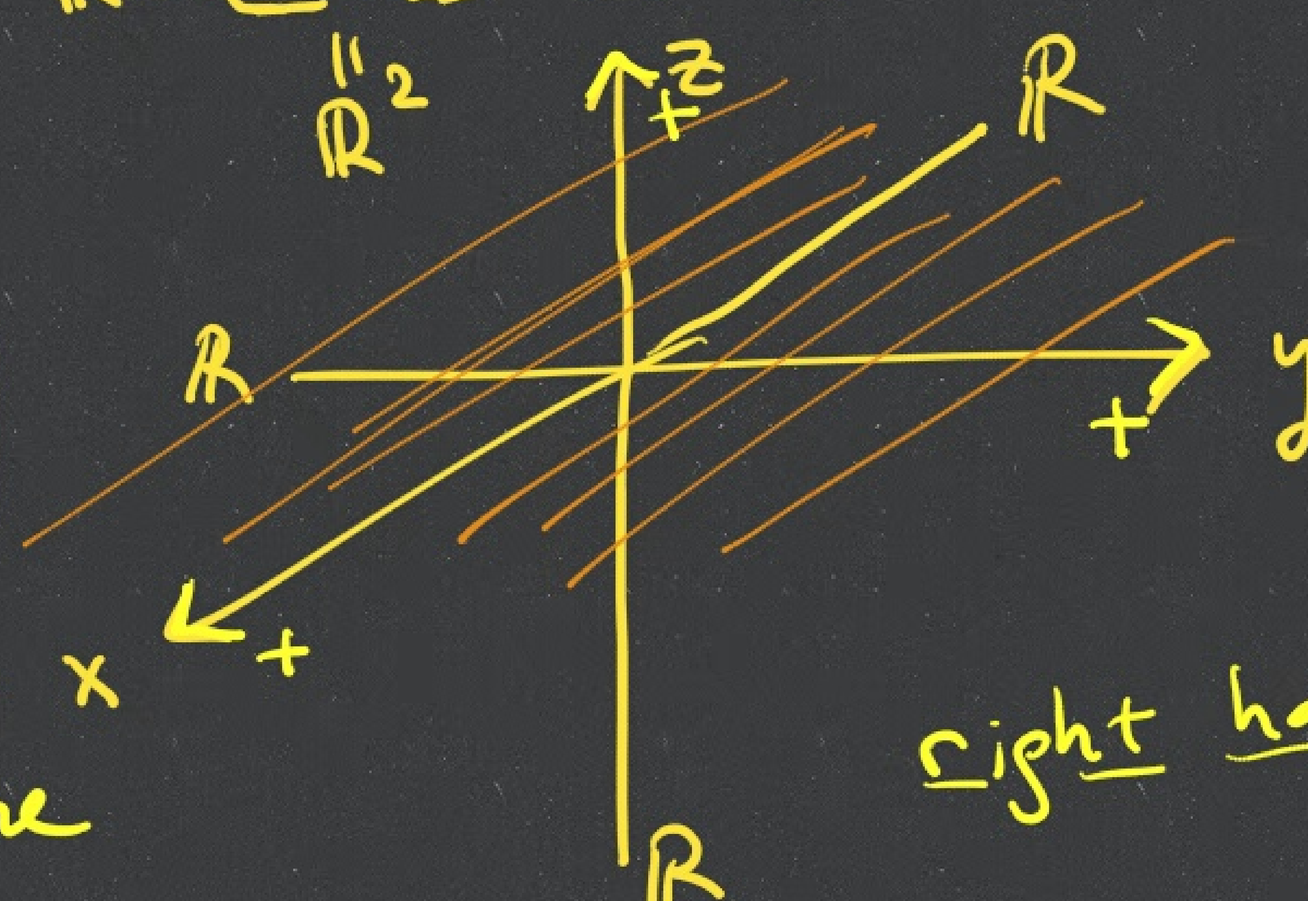
- origin in $\mathbb{R}, \mathbb{R}^2, \mathbb{R}^3$
- xy-plane ----
- yz-plane
- xz-plane
- right handed orientation

$\mathbb{R}$

$$\mathbb{R}^2=\mathbb{R}\times\mathbb{R}=\{(a,b)\mid a\in\mathbb{R},\ b\in\mathbb{R}\}$$



$$\mathbb{R}^3=\underbrace{\boxed{\mathbb{R}\times\mathbb{R}}}_{=\mathbb{R}^2}\times\mathbb{R}=\{(a,b,c)\mid a,b,c\in\mathbb{R}\}$$



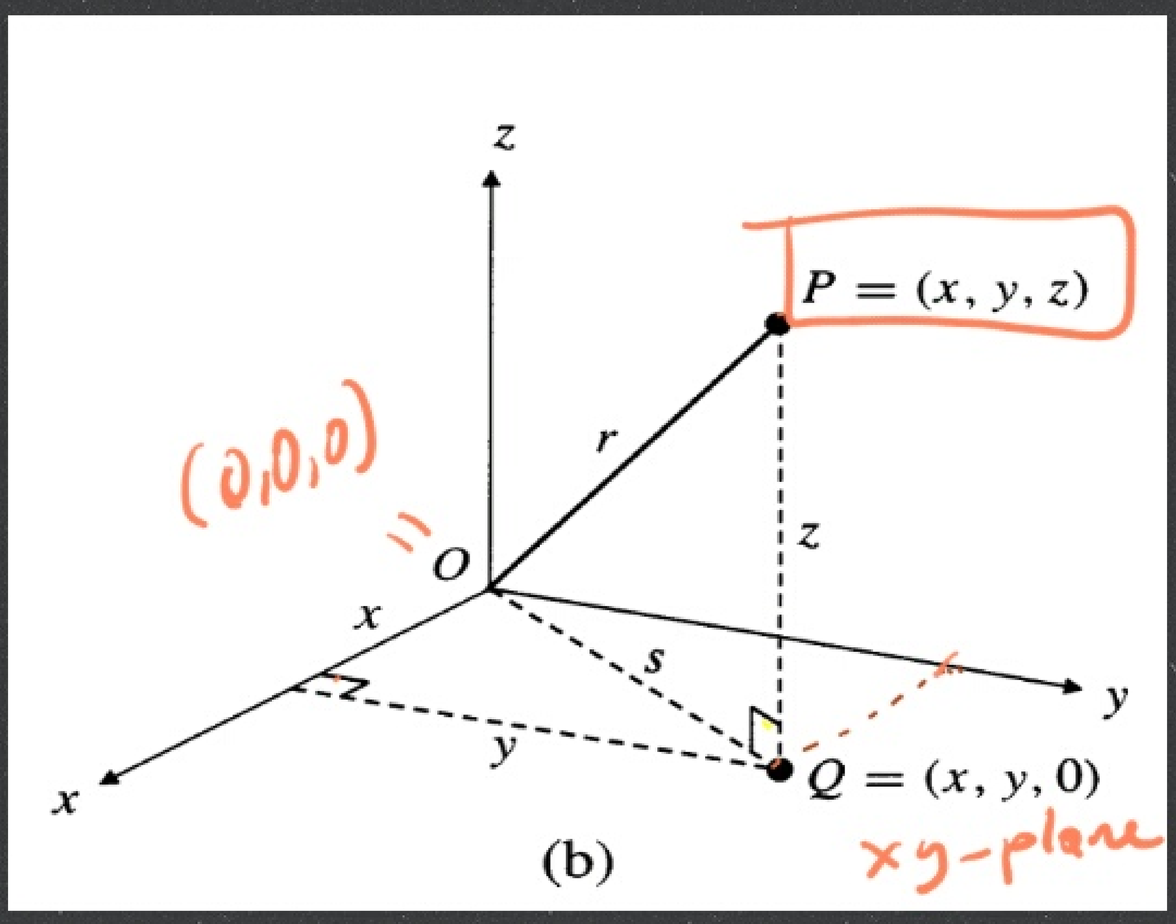
right handed orientation

$z=0$ is the xy-plane

$y=0$ is the xz-plane

$x=0$ is the yz-plane

r = Distance of a point $P=(x,y,z)$ to the origin O $= (0,0,0)$



(b)

$$s^2 = x^2 + y^2$$

$$r^2 = s^2 + z^2 \Rightarrow r^2 = x^2 + y^2 + z^2$$

$$\Rightarrow r = \sqrt{x^2+y^2+z^2}.$$

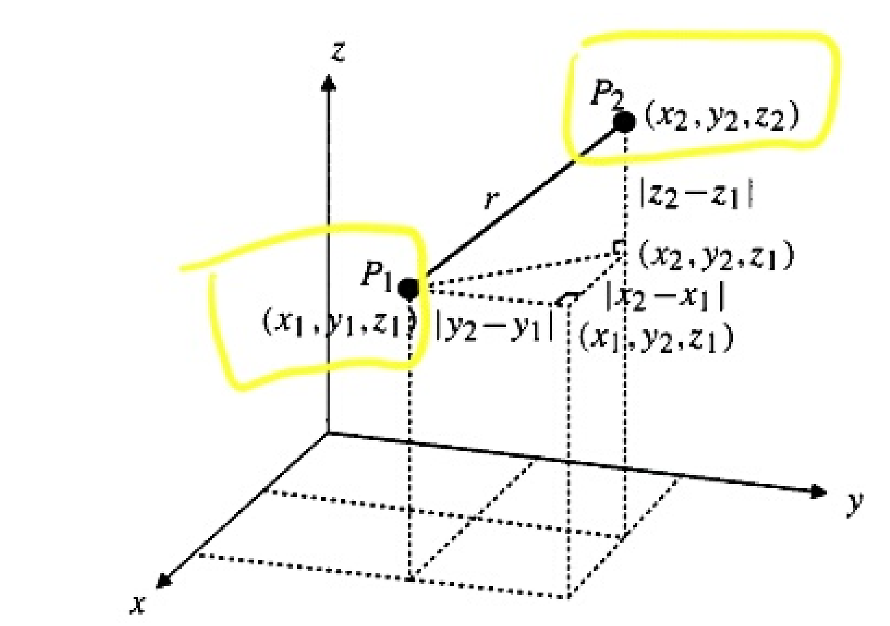


Figure 10.2 Distance between points

Distance between two points in $\mathbb{R}^3$

$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$ is the distance between $P_1 = (x_1, y_1, z_1)$, $P_2 = (x_2, y_2, z_2)$.

$$r^2 = (x-a)^2 + (y-b)^2 + (z-c)^2 \quad (*)$$

$(x, y, z) \in$ Sphere $\Rightarrow (x, y, z)$ satisfies $(*)$

Equation of a sphere of radius r centered at (a, b, c) is