ex: $E(x) = \int_0^x e^{-t^2}\,dt$. Find the Maclaurin series for $E(x)$.

$$e^t = \sum_{n=0}^{\infty} \frac{t^n}{n!} \quad \text{for all } t \in (-\infty, \infty)$$

$$e^{-t^2} = \sum_{n=0}^{\infty} \frac{(-t^2)^n}{n!} = \sum_{n=0}^{\infty} \frac{(-1)^n t^{2n}}{n!} \quad \text{for all } t \in (-\infty, \infty)$$

$$E(x) = \int_0^x e^{-t^2}\,dt = \int_0^x \sum_{n=0}^{\infty} \frac{(-1)^n t^{2n}}{n!}\,dt$$

$$= \sum_{n=0}^{\infty} \left(\int_0^x \frac{(-1)^n t^{2n}}{n!}\,dt \right) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)\cdot n!}$$

for all $x \in (-\infty, \infty)$

ex: Calculate $E(1)$ correct to within an error of 0.001. $= \frac{1}{1000}$

$$E(1) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)\cdot n!} = 1 - \frac{1}{3} + \frac{1}{5\cdot 2!} - \frac{1}{7\cdot 3!} + \frac{1}{9\cdot 4!} - \frac{1}{11\cdot 5!} \cdots$$

$+ \quad - \quad + \quad - \quad + \quad -$

This is an alternating series, satisfies (i)-(iii) of AST. i) $a_n \cdot a_{n+1} < 0$ ✓

ii) $|a_{n+1}| \leq |a_n|$ ✓

iii) $\lim_{n \to \infty} a_n = 0$ ✓

$$E(1) = \underbrace{1 - \frac{1}{3} + \overset{\frac{1}{10}}{\frac{1}{5 \cdot 2!}} - \overset{\frac{1}{42}}{\frac{1}{7 \cdot 3!}} + \overset{\frac{1}{216}}{\frac{1}{9 \cdot 4!}}} - \overset{\frac{1}{1320}}{\frac{1}{11 \cdot 5!}} \cdots$$

By the error estimate for AST, we have

$$E(1) \approx \boxed{1 - \frac{1}{3} + \frac{1}{10} - \frac{1}{42} + \frac{1}{216}} \quad \text{(five terms)}$$

with $|\text{error}| \leq \dfrac{1}{1320} < \dfrac{1}{1000} = 0.001$

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ex: $$\lim_{x \to 0} \frac{x - \sin x}{x^3} = \lim_{x \to 0} \frac{\cancel{x} - \left(\cancel{x} - \frac{x^3}{3!} + \frac{x^5}{5!} \cdots\right)}{x^3}$$

$$= \lim_{x \to 0} \frac{\frac{x^3}{3!} - \frac{x^5}{5!} - \cdots}{x^3} = \lim_{x \to 0} \frac{\frac{1}{3!} - \frac{x^2}{5!} \cdots}{1} = \frac{1}{3!} = \frac{1}{6}$$

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ex: $$\lim_{x \to 0} \frac{(e^{2x} - 1)\ln(1 + x^3)}{(1 - \cos 3x)^2}$$

$$= \lim_{x \to 0} \frac{\left(\cancel{1} + (2x) + \frac{(2x)^2}{2!} + \frac{(2x)^3}{3!} \cdots \cancel{-1}\right) \cdot \left(x^3 - \frac{x^6}{2} \cdots\right)}{\left(\cancel{1} - \left(\cancel{1} - \frac{(3x)^2}{2!} + \frac{(3x)^4}{4!} - \cdots\right)\right)^2}$$

$$= \lim_{x \to 0} \frac{\left(2x + 2x^2 + \frac{8}{3}x^3 \ - - -\right) \cdot \left(x^3 - \frac{x^6}{2} \ - - -\right)}{\left(\frac{9}{2}x^2 - \frac{81}{24}x^4 \ - - -\right)^2}$$

$$= \lim_{x \to 0} \frac{2\cancel{x^4} + 2\cancel{x^5}^{\,x} \ - - -}{\cancel{x^4} \cdot \left(\frac{9}{4} - \frac{81}{24}x^2 \ - -\right)^2}$$

$$= \lim_{x \to 0} \frac{2 + 2x + - - -}{\left(\frac{9}{4} - \frac{81}{24}x^2 \ - - -\right)^2} = \frac{2}{\left(\frac{9}{2}\right)^2} = 2 \cdot \frac{4}{81}$$

$$= \frac{8}{81}$$